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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. END SEMESTER ARREAR EXAMINATION, MAY 2024

MATHEMATICS [GENERAL]

Paper : II

Date : 29/05/2024

Time : 11 am – 2 pm

Full Marks : 75

[Use a separate Answer Book for **each group**]

Group-A

Answer **any five** questions: [5×5]

1. Find the differential equation of all circles having constant radius a. [5]
2. Solve : $ydx + xdy + ln\,x\,dx = 0$. [5]
3. Solve : $\left(x^2y^3 + 2xy\right)dy = dx$. [5]
4. Solve : $xy - \frac{dy}{dx} = y^3 e^{-x^2}$. [5]
5. Obtain the complete primitive and singular solution of the following Clairaut's equation.
   $y = px + p - p^2$ [5]
6. Solve : $\frac{d^2y}{dx^2} + y = 0$ given $y = 2$ for $x = 0$
   and $y = -2$ for $x = \frac{\pi}{2}$. [5]
7. Solve : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = log\,x$. [5]
8. Find the equation of orthogonal trajectories of the family of curves
   $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda^2} = 1$
   where a is a constant and λ is a parameter. [5]

Group-B

Answer **any five** questions: [5×10]

9. a) Show that the sequence $\sqrt{2}, \sqrt{2+\sqrt{2}}, \sqrt{2+\sqrt{2+\sqrt{2}}}$,.... tends to a definite finite limit and find the limit.
   b) State Cauchy's general principle of convergence and use it to prove that the sequence $\{x_n\}$ given by $x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$ converges. [5+5]

10. a) Find whether the series is convergent or divergent:
    $x^2 + \frac{2^2}{3.4}x^4 + \frac{2^2.4^2}{3.4.5.6}x^6 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8 + ...\ (x > 0)$
    b) Test the convergence of the series Σu_n, where $u_n = \frac{n^n}{(n+1)^{n+1}}$. [5+5]

11. a) Show that $\lim_{x\to 0} cos\left(\frac{1}{x}\right)$ does not exist.
    b) Check the continuity of the following function at $x = 0$
    $$f(x) = \begin{cases} \frac{tan^2 x}{3x}; & x \neq 0 \\ 2/3\ ; & x = 0 \end{cases}.$$ [6+4]

12. a) If $y = e^{a \sin^{-1} x}$, prove that

$$\left(1-x^2\right) y_{n+2} - (2n+1) x y_{n+1} - \left(n^2 + a^2\right) y_n = 0 \text{, where } y_n = \frac{d^n y}{dx^n} .$$

b) Find a and b such that $\lim_{x \to 0} \frac{x(1 + a\cos x) - b \sin x}{x^3} = 1$. [5+5]

13. a) If $\tan u = \frac{x^3 + y^3}{x - y}$ prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x dy} + y^2 \frac{\partial^2 u}{\partial y^2} = \left(1 - 4\sin^2 u\right) \sin 2u .$$

b) Let $f(x,y) = \begin{cases} xy & \text{if } |x| \ge |y| \\ -xy & \text{if } |x| < |y|. \end{cases}$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$. [5+5]

14. a) Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$.

b) Find the asymptotes of the curve
$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$. [5+5]

15. a) Evaluate $\iint \left(x^2 + y^2\right) dxdy$ over the region bounded by $xy = 1,\ y = 0,\ y = x$ and $x = 2$.

b) Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1). [7+3]

16. a) Find the envelope of the family of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a + b = c$ (c is a constant.)

b) The circle $x^2 + y^2 = a^2$ revolves about x axis. Find the volume of the sphere thus generated. [5+5]

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